USN | | | | | | | | | | |

MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2015/Jan.2016

Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the direction cosines of the line which is perpendicular to the lines with direction cosines (3, -1, 1) an (-3, 2, 4). **(06 Marks)**

b. If $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the direction cosines of a line, then prove the following:

i) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

ii) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ **(07 Marks)**

c. Find the projection of the line AB on the line CD where A = (1, 2, 3), B = (1, 1, 1), C = (0, 0, 1), D = (2, 3, 0). **(07 Marks)**

2 a. Find the equation of the plane through (1, -2, 2), (-3, 1, -2) and perpendicular to the plane $2x - y - z + 6 = 0$. **(06 Marks)**

b. Find the image of the point (1, -2, 3) in the plane $2x + y - z = 5$. **(07 Marks)**

c. Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. **(07 Marks)**

3 a. Find the constant 'a' so that the vectors $2i - j + k$, $i + 2j - 3k$ and $3i + aj + 5k$ are coplanar. **(06 Marks)**

b. Prove that $\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right] = 2\left[\vec{a}, \vec{b}, \vec{c}\right]$. **(07 Marks)**

c. Find the unit normal vector to both the vectors $4i - j + 3k$ and $-2i + j - 2k$. Find also the sine of the angle between them. **(07 Marks)**

4 a. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ where t is the time. Find the components of its velocity and acceleration at time t = 1 in the direction of $2i + 3j + 6k$. **(06 Marks)**

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x = z^2 + y^2 - 3$ at the point (2, -1, 2). **(07 Marks)**

c. Find the directional derivative of $\phi = xy^2 + yz^3$ at the point (1, -2, -1) in the direction of the normal to the surface $x\log z - y^2 = -4$ at (-1, 2, 1). **(07 Marks)**

5 a. Prove that $\text{div}(\text{curl}\,\vec{A}) = 0$. **(06 Marks)**

b. Find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

c. Show that the vector $\vec{F} = (3x^2 - 2yz)i + (3y^2 - 2zx)j + (3z^2 - 2xy)k$ is irrotational and find ϕ such that $\vec{F} = \text{grad}\,\phi$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator, will be treated as malpractice.

6 a. Find: $L\{\cos t\ \cos 2t\ \cos 3t\}$. **(06 Marks)**

b. Find: i) $L\{e^{-t}\cos^2 t\}$, ii) $L\{te^{-t}\sin 3t\}$. **(07 Marks)**

c. Find: $L\left\{\frac{\cos at - \cos bt}{t}\right\}$. **(07 Marks)**

7 a. Find: $L^{-1}\left\{\frac{4s+5}{(s-1)^2(s+2)}\right\}$. **(06 Marks)**

b. Find: i) $L^{-1}\left\{\frac{s+2}{s^2-4s+13}\right\}$, ii) $L^{-1}\left\{\log\left(\frac{s+a}{s+b}\right)\right\}$. **(07 Marks)**

c. Find: $L^{-1}\left\{\frac{1}{s^2(s+1)}\right\}$. **(07 Marks)**

8 a. Using Laplace transforms, solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^{2t}$ with $y(0) = 0$, $y'(0) = 1$. **(10 Marks)**

b. Using Laplace transformation method solve the differential equation $y'' + 2y' - 3y = \sin t$, $y(0) = y'(0) = 0$. **(10 Marks)**

* * * * *